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THEORETICAL STUDIES OF SOME NONLINEAR ASPECTS OF HYPERSONIC PANEL FLUTTER

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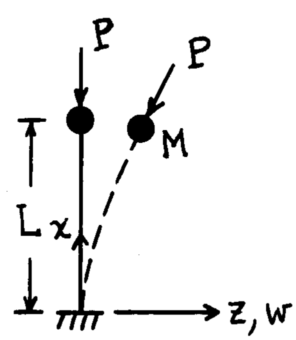
Project Status

METHODS OF SOLUTION:

In accordance with the research proposal, two specific pilot problems have been investigated, one illustrating the application of a variational principle to a so-called nonconservative system and the other demonstrating an inherent weakness of the Galerkin procedure when it is applied to the solution of differential equations in a purely mathematical sense.

To illustrate that a variational formulation of a so-called nonconservative-system problem can be carried out, the classical problem of the follower-force column-buckling problem (See Fig. 1) is undertaken. The classical solution, found by differential-equation solution, is given by Bolotin in Reference 1.



igure 1. Column with Follower Force

Consider the functional

$$F = \frac{EI}{2}\int_0^L \left(\frac{d^2w}{dx^2}\right)^2 dx - \frac{P_x}{2}\int_0^L \left(\frac{dw}{dx}\right)^2 dx - \frac{M}{2}\omega^2 (w)^2_{x=L} + P_z (w)_{x=L}$$

where

EI = Uniform-beam bending stiffness

w = Lateral deflection

M = Mass density of end mass

ω = Natural frequency of vibration of beam

P = Follower force

P_x, P_z = Axial and lateral components, respectively, of follower force

and the mass density of the beam itself has been neglected.

With P_x and P_z held constant during the first variation of F with respect to w, the following Euler equations and natural boundary conditions result:

$$EI\frac{d^4w}{dx^4} + P_x\frac{d^2w}{dx^2} = 0 \qquad 0 < x < L$$

$$-EI\frac{d^3w}{dx^3} - M\omega^2 w - P_x\frac{dw}{dx} + P_z = 0 \qquad \text{at } x = L$$

$$EI\frac{d^2w}{dx^2} = 0 \qquad \text{at } x = L$$

But for small oscillations

$$P_x \simeq P, \quad P_z \simeq P\left(\frac{dw}{dx}\right)_{x=L}$$

Thus, in final form, the results are:

$$EI\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} = 0$$

$$EI\frac{d^3w}{dx^3} + M\omega^2 w = 0 \qquad \text{at } x = L$$

$$EI\frac{d^2w}{dx^2} = 0 \qquad \text{at } x = L$$

$$\text{for} \quad w = \frac{dw}{dx} = 0 \qquad \text{at } x = 0$$

The above equations, stemming from application of the indirect method of the calculus of variations to the assumed functional F, lead to the governing equations and boundary conditions given by Bolotin.

With confidence established in the functional F, the direct method of the calculus of variations is employed to seek the critical value of P. The geometric constraints are satisfied with

$$w = \sum_{odd}^{\infty} a_n \left[1 - \cos\frac{n\pi x}{2L}\right]$$

Insertion of w into F, integration of the results, and subsequent variation with respect to a_n while P_x and P_z are held constant leads to a condition for ensuring the existence of the lateral-deflection amplitudes at buckling, namely,

$$\bar{M} J_2 \bar{P} J_1 - [1 - \bar{M} J_1][1 - \bar{P} J_2] = 0$$

where

$$\bar{M} = \frac{32}{\pi^4}\frac{M\omega^2 L^3}{EI}, \qquad J_1 = \sum_{odd}^{\infty} \frac{1}{n^2(n^2-k^2)},$$

$$\bar{P} = \frac{PL^2}{EI}, \qquad J_2 = \sum_{odd}^{\infty} \frac{n \sin\frac{n\pi}{2}}{n^2(n^2-k^2)}$$

and

$$k^2 = \frac{4}{\pi^2}\frac{PL^2}{EI}$$

From the characteristic equation, it is easily shown that the frequency grows without bound when

$$\frac{\pi k}{2} = \tan\frac{\pi k}{2}$$

or

$$\sqrt{\frac{PL^2}{EI}} = \tan\sqrt{\frac{PL^2}{EI}}$$

The lowest nontrivial root of this equation is

$$\sqrt{\frac{PL^2}{EI}} = 4.49 \quad \text{or} \quad P_{cr} = 20.2\,\frac{EI}{L^2}$$

which is identical to the result given by Bolotin.

As an illustration of an inherent weakness of the Galerkin procedure when applied to the solution of differential equations in more than simply a mathematical sense, the equation for a harmonically vibrating Timoshenko beam (Reference 2) is solved for the case of clamped ends (see Fig. 2).

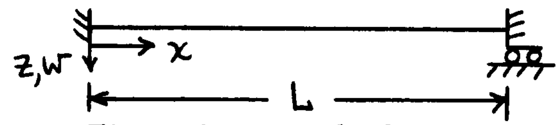


Figure 2. Timoshenko Beam

The Timoshenko-beam equation can be written, neglecting rotary-inertia effects, as

$$EI\frac{d^4 w}{dx^4} + \frac{EI}{D_Q}\rho\omega^2\frac{d^2 w}{dx^2} - \rho\omega^2 w = 0$$

where

EI = Beam bending stiffness

w = Lateral deflection

D_Q = Beam transverse-shear stiffness parameter

ρ = Mass density of beam

ω = Natural frequency of beam vibration

Let

$$w = A\left[1 - \cos\frac{2\pi x}{L}\right]$$

so that all boundary conditions (in this case all geometric in nature) are satisfied. The Galerkin integral for determining the existence of the amplitude parameter A is

$$\int_0^L \left\{-EIA\left(\frac{2\pi}{L}\right)^4\cos\frac{2\pi x}{L} + \frac{EI}{D_Q}\rho\omega^2 A\left(\frac{2\pi}{L}\right)^2\cos\frac{2\pi x}{L} - \rho\omega^2 A\left(1-\cos\frac{2\pi x}{L}\right)\right\}\left(1-\cos\frac{2\pi x}{L}\right)dx = 0$$

If, indeed, A exists, the eigenvalue is

$$\omega^2 = \frac{\frac{16\pi^4}{3}\frac{EI}{\rho L^4}}{\left(1 + \frac{4}{3}\frac{\pi^2 EI}{D_Q L^2}\right)} \simeq \frac{22.8\,\frac{EI}{\rho L^4}}{\left(1 + \frac{4}{3}\frac{\pi^2 EI}{D_Q L^2}\right)}$$

Now, if the same problem is approached via the potential-energy method, the total potential of the problem includes the energy of bending, the energy of shearing, and the kinetic energy of vibration. Thus,

$$U - T = \frac{EI}{2}\int_0^L \left[\frac{d^2w}{dx^2} - \frac{1}{D_Q}\frac{\partial Q}{\partial x}\right]^2 dx + \frac{1}{2}\int_0^L \frac{Q^2}{D_Q} dx - \frac{1}{2}\rho\omega^2 \int_0^L w^2 dx$$

where Q is the shearing force on the beam cross section. The direct method of the calculus of variations (Rayleigh-Ritz method) requires that functions satisfying the geometric boundary conditions be selected when the functional to be minimized is the total potential energy; thus, the degrees of freedom are taken as

$$w = A\left[1 - \cos\frac{2\pi x}{L}\right]$$

$$Q = B\sin\frac{2\pi x}{L}$$

Actually, Q is the shear force, but since $Q/D_Q = \gamma$ is the shear angle, a direct proportion exists and Q may be used interchangeably with γ.

Now, enforcement of the condition $\delta(U-T) = 0$ with respect to both A and B yields two simultaneous equations for ensuring the existence of A and B. The resulting characteristic equation yields the eigenvalue

$$\omega^2 = \frac{\frac{16}{3}\pi^4 \frac{EI}{\rho L^4}}{\left(1 + 4\frac{\pi^2}{L^2}\frac{EI}{D_Q}\right)}$$

It is to be noticed that this result, which represents a minimum-energy solution, is different from the Galerkin solution, which represents only an approximate solution to a differential equation. The reason for this discrepancy is that the Timoshenko-beam equation is not an Euler equation. Interestingly enough, in the case of simple supports, no difference exists in the two methods of solution; however, for clamped supports, a Galerkin solution yielding the same results as the minimum-energy solution can only be achieved if the two Euler equations governing the problem are utilized. In the present problem, the discrepancy is serious since the Galerkin solution underestimates the effect of transverse shear. The problem is very serious in the case of sandwich beams, where the transverse shear is extremely important.

The foregoing pilot problems indicate that continuing results on more complicated problems may lead to reevaluation of the methods and results associated with non-conservative nonlinear problems having boundary restraints other than simple supports.

EFFECTS OF AERODYNAMIC NONLINEARITIES:

Bolotin et al.[3] considered a two-dimensional simply supported plate with aerodynamic loads from second-order piston theory and variable in-plane edge restraint. Aerodynamic damping terms were left out. They found that, for certain values of this edge restraint, self-sustained oscillations of the plate were possible for Mach numbers below the critical Mach number from linear theory, if the initial disturbance of the plate was large enough. Furthermore, the disparity between the two Mach numbers increased as the magnitude of the initial disturbance increased. However, the values of the parameters used in the analysis were somewhat removed from those found in current practice, so one might conclude that such a problem was only of academic interest.

On the other hand, Stepanov[4] studied a three-dimensional simply supported plate with infinite in-plane edge restraint and second-order piston-theory aerodynamic loads including some aerodynamic damping terms. The values of the parameters he used were far more representative of current practice, and he obtained the curious result that the critical velocity of the plate became greater than that obtained from linear theory as the magnitude of the initial disturbance increased.

It would, therefore, appear useful to examine further the question of the influence of initial deformations on the critical speed. This will be accomplished as part of an evaluation of the effects of aerodynamic nonlinearities by comparison with Dowell's[5] solutions. The applicable equations are being programmed for the computer, and numerical results will soon be available.

ANNUAL REPORT: An annual report on the first year's activities has been prepared and will be distributed.

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Principal Investigator

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4. Stepanov, R. D., "Problems of Plate Flutter in Nonlinear Arrangement", FTD TT 64-596 (1965); translated from Izv. Vyssh. Ucheb. Zav., Mashino., No. 12, 46-54 (1960).

5. Dowell, E. H., "Nonlinear Oscillations of a Fluttering Plate," AIAA J. 4, 1267-1275 (1966).